AN OPTIMIZATION METHOD FOR LARGE-SCALE 3D PRINTING

Generate external axis motion using Fourier series

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Abstract. With the increase in labor costs, more and more robot constructions appear in building construction and spatial structure fabrication. There are many robots working on large-scale objects. When the reach range of the robot cannot meet the requirements, so an external axis is needed. The external axis is usually a linear motion device, which can significantly increase the operating range of the robotic arm. In actual construction, it is also widely used. This article introduces a 3d printing coffee bar project. Because this project is of a large scale and needs to be printed at one time, the XYZ external axis was used in this project to complete the task. Inspired by this project, this article study several methods of optimizing the motion of external axes in large-scale construction. Finally, we chose to use the Fourier series as the most suitable method to optimize the printing path and programed this method as a component of FUROBOT for more convenient use. This article explains the principle of this method in detail. Finally, this article uses a 3D printing example to illustrate the precautions in actual use.

Keywords. Robotics; motion optimize; Fourier series; 3D printing; external axis.

1. Introduction

With the increase in labor costs, more and more robot constructions appear in build-ing construction (Chai,et al.2019)(Chen,et all.2019).There are a large part of robots are working for a large scale objects. When the reach of the robot arm cannot meet the requirements, you need to rely on the external axis for work. The architectural fabrication external axis is usually a linear motion device, which can greatly increase the operating range of the robotic arm. In actual construction, it is also widely used. This article introduces a real printed coffee bar (Figure 1), because this project has a large scale and needs to be printed at one time, so the xyz external axis was used in this project to complete the task. Inspired by

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this project, this article begins to study the problem of optimizing the motion of large-scale construction assisted by external axes.

[a] [b]

Figure 1. coffee bar 3D-printing.

In the printing process using external axis, the movement of external axis can be controlled independently. Generally speaking, the programmer is required to formu-late the movement trajectory of the external axis, but if you encounter a large and complex project, this work is time-consuming and labor-intensive, and the manually planned movement path of the external axis is not necessarily reasonable. How to automatically generate the external axis motion path and generate a better path, these two problems are trying to solve this article.

2. Methodology

Usually, we use the spatial coordinate information of the target point to program the external axis. This example uses an object that is not very large for printing, for the convenience of demonstration (Figure 2).The robot is located on a single external axis.In this example, the biggest movement direction is y-axis.We need to complete the motion programming of the external axis in the y-axis direction.

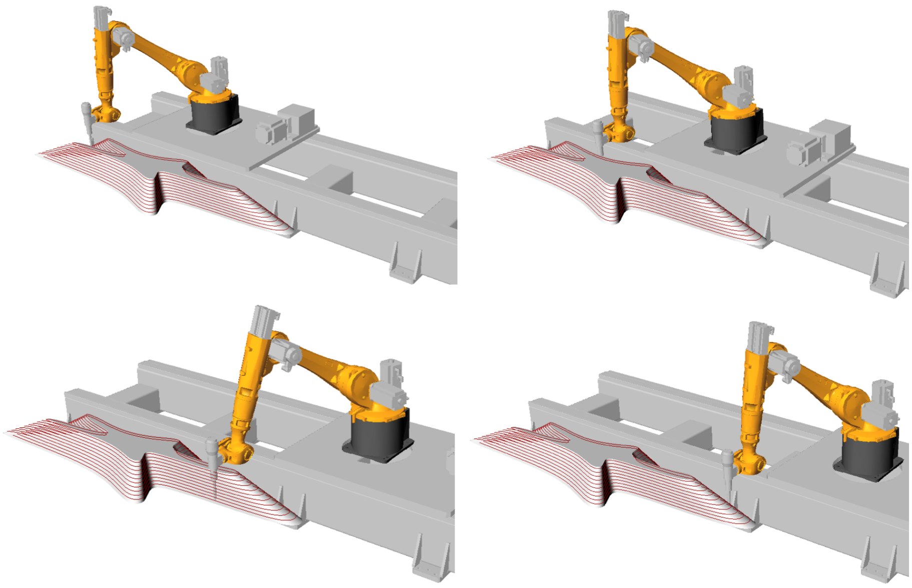

Figure 2. Simulation for 3D printing.

Programming directly with the target point data (Figure 3a), any change in the target point will directly affect the movement of the external axis, so it causes two problems:

- If there is a large and high-frequency position change between the target point and the point, it will cause vibration of the external axis, resulting in possible devia-tions in the work of the robot arm.
- Moving completely according to the target point will make the external axis move unnecessarily, and the movement will cause sway, so it is a more reasonable solu-tion to minimize the movement of the external axis and increase the motion of the robot to compensate.

If you use a manual method to straighten the motion curve, then some small excess motion will be drawn into a straight line and thus be deleted, so this method will produce a smoother path than the previous method. But the biggest problem with this approach is that it requires human participation (Figure 3b). At the same time, when the path undergoes the same drastic change, the external axis movement will also produce sharp abrupt changes. Of course, this problem can also be solved by manual adjustment again. This method is labor intensive, and this shortcoming limits the intelligent use of this method in general printing small builds.

So this article considers to solve the above problems directly from curve fitting. (Figure 3c).

Polynomial fitting can achieve this goal, , this method has ability to generate smooth curves, and easy to generate in algorithm and code. But the disadvantages obviously: this method is not suitable for generating periodic curves. If we increase the number of cycles of the curve, in order to better fit the target curve, we have to increase the number of polynomials accordingly, which will bring a certain burden to the calculation and even cause memory overflow when programming. In actual print-ing, the number of cycles is the number of layers printed, generally there are thou-sands of layers, which is impractical to use polynomials.The same property exist in Bezier curve.

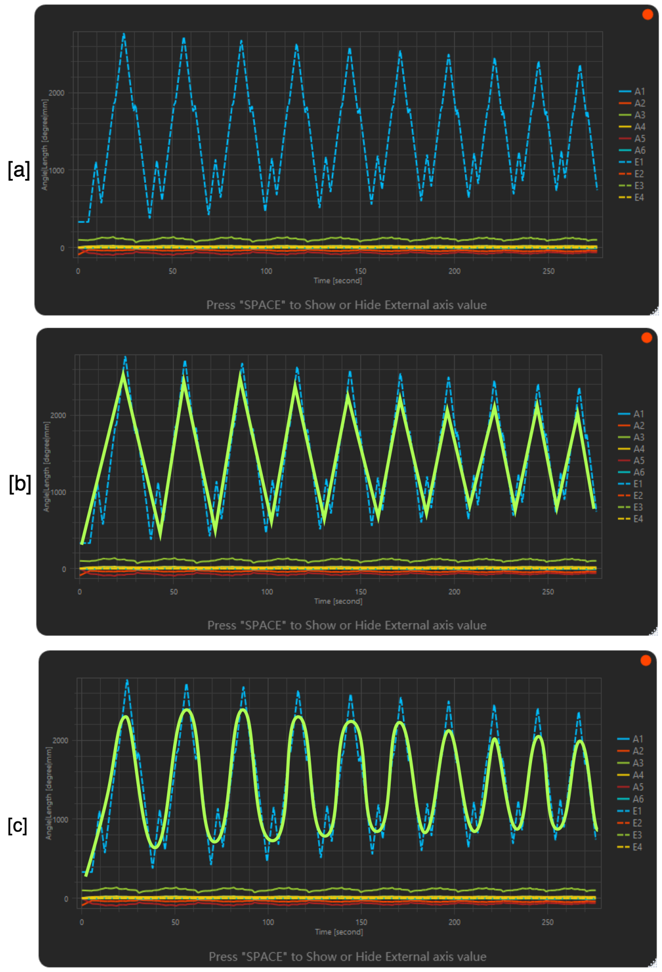

Figure 3. External axis visualization.[a]:Using target plane position .[b]: Using straight line to fit.[c]: Using a smooth curve to fit.

We need a method that can fit periodic curves,so we chose the fourier series(Lay,et al.2015).

2.1. PROJECTION

Before introducing Fourier series,it is necessary to talk about projection.

If there are two vector a and vector b, we need to find a coefficient so that after the coefficient is multiplied by the a vector, it can be as close to the b vector as possible. According to linear algebra, the following formula (1) can be used to find this scale coefficient:

$$c = \frac{b \cdot a}{a \cdot a} \tag{1}$$

So if multiply c to a, we can get $\hat{a} = c \cdot a$(Figure 4):

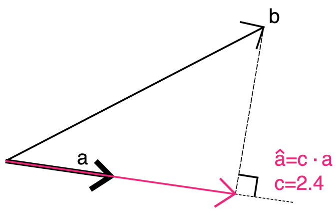

Figure 4. Projection.

This process is called projection.We can make an analogy between vector and function. We project the B function onto the A function and find a coefficient C such that the A function multiplied by C becomes the projection of the B function on the A function:

$$C = \frac{\langle B, A\rangle}{\langle A, A\rangle} \tag{2}$$

The expression $\langle A, B\rangle$ is called the inner product of the function A and B.As we all know,the definition of inner product of 2 vectors is:

$$v_1 = [x_1, y_1, z_1]^T, v_2 = [x_2, y_2, z_2]^T \tag{3}$$

$$v_1 \cdot v_2 = x_1 \cdot x_2 + y_1 \cdot y_2 + z_1 \cdot z_2 \tag{4}$$

As an anology,we define the inner product of one function $\langle A, B\rangle$ as:

$$\langle A, B\rangle = \int_a^b A(x)B(x)\,dx \tag{5}$$

We only need to find out the coefficients of our existing B function for several si-ne and cosine functions of different frequencies, and then we can use these sine and cosine functions to assemble this B function.

2.2. FOURIER SERIES

The intention of Fourier series is to decompose an arbitrary function into sine and cosine functions of different frequencies. With the helps of certain coefficients,all these trigonometric functions take a certain independent variable(such as x) and the sum of these harmonic functions is approximately equal to original function.

Firstly,we define the domain of harmonic function belongs to [0,2π],but in actual 3D printing,the number of target points are very large,usually tens of thousands of points.In order to be able to use this method,we first need to scale all target points to the interval of [0,2π].

We use this formula to coefficient:

$$\text{coeff} = \frac{2\pi}{\text{NumberOfPts} - 1} \tag{6}$$

Because fourier series consists of two parts, the cosine part and the sine part, we take the sine function as an example. Because we need to solve the Fourier series within[0,2π], we need to multiply the independent variable by the scale coefficient obtained in (6):

$$t = (\text{IndexOfPts}) \cdot \text{coeff} \tag{7}$$

We assume that there is a function: $\cos(t)t[0, 2\pi]$.If we divide $[0, 2\pi]$ into 5 equal parts,then we will have 6 points: $t_1 = 0, t_2 = \frac{2\pi}{5}, t_3 = \frac{4\pi}{5}, t_4 = \frac{6\pi}{6}, t_5 = \frac{8\pi}{5}, t_6 = \frac{10\pi}{5} = 2\pi$,and $d_t = \frac{2\pi}{5} = 1.2566$.

Add up each of them:$\cos(t_1) \cdot \cos(t_1) + \cos(t_2) \cdot \cos(t_2) + \cos(t_3) \cdot \cos(t_3) + \ldots + \cos(t_6) \cdot \cos(t_6) = \pi$

If we divide the domain into infinite small segments d_t,we also can get:

$$\int_0^{2\pi} \cos(t) \cdot \cos(t)\, dt = \langle \cos(t), \cos(t) \rangle = \pi, t > 0 \tag{8}$$

There exist strict proof of(8).Formula (8) represent inner product of function cosine.The same way to do sine function ,we can get:

$$\int_0^{2\pi} \sin(t) \cdot \sin(t)\, dt = \langle \sin(t), \sin(t) \rangle = \pi, t > 0 \tag{9}$$

We define:

$$a_n = \frac{1}{\pi} \sum_{t=0}^{2\pi} f(t) \cos(n \cdot t)\, dt\, , b_n = \frac{1}{\pi} \sum_{t=0}^{2\pi} f(t) \sin(n \cdot t)\, dt\, , n > 0 \tag{10}$$

From formula (12),when n=0,we can get: $\langle \cos(0), \cos(0) \rangle = 2\pi$ rather than π,therefore we must divide this coefficient of cosine part by 2, which is $\frac{a_0}{2}$,and the coefficient of sine doesn't exist when n=0.We can conclude:

$$f(x) = \frac{a_0}{2} + a_1 \cos(t) + b_1 \sin(t) + a_2 \cos(2t) + b_2 \sin(2t) + \ldots + a_n \cos(n \cdot t) + b_n \sin(n \cdot t) \tag{11}$$

Function (11) is the final function assembled by several sine and cosine function with different frequency.This function is also continuous,but in real case of 3D-printing,each data point is discrete,so we need to change the integral into a summation.

Harmonic function itself is a periodic function, which satisfies our need for long-term periodic motion curve fitting. And you can set different orders for the Fourier series to obtain different smoothing effects (Figure 5). The order is actually "n" in function (11), which represent the highest frequency of the sine function and cosine function participating in the entire Fourier series.

After programming this method, the burden on the computer's cpu is small. After actual testing, the final calculation result can be obtained in real time.

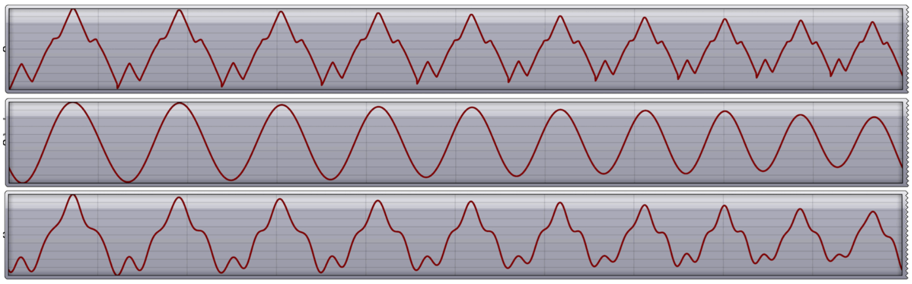

Figure 5. Top:original input sequence , Middle:order=15 ,Bottom:order=50.

3. Instructions

Because it is difficult for architects to apply these mathematical algorithms in their work, we wrote the above algorithms as a grasshopper component, which is built into FUROBOT for people to download and use for free.In specific usage, we only need to separate the xyz components of the target point separately by using "decompose point" component, and then find the Fourier series for each component. Then, on the obtained Fourier series, we can manually add an offset so that the robot is within the printing range of the object.

In detail, the two aspects of this paper that need to be solved:

1. Keep the external axis as stable as possible to avoid sudden curvature changes.
2. Minimize the length of the external axis movement path.

While satisfying these two requirements, it is necessary to ensure that the entire printing process does not occur such as joint limit, exceeding the printing range, etc., if the above phenomenon occurs, increase the order.

3.1. EXAMPLE

One example of the use of this method is demonstrated (Figure6), which uses XYZ three-axis external axis. This example verified by simulation and proved that this method is feasible and effective. Users can adjust the order number to get a smoother path curve, then check the reachability of TCP by this the path.

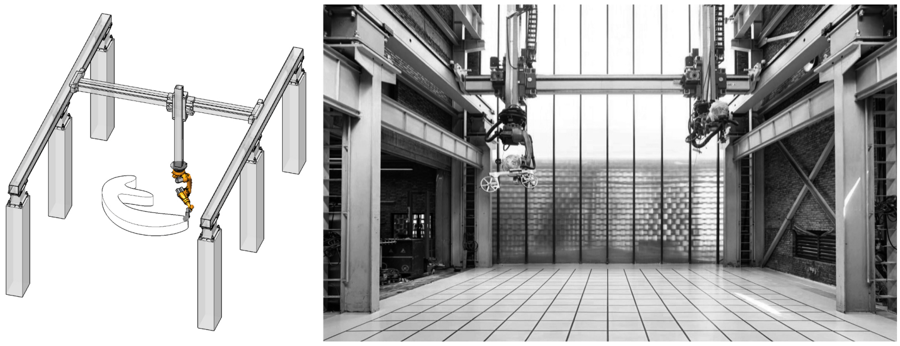

Figure 6. XYZ external axis.

In this example, first we have to find the smallest order, because the smaller the order, the better it can meet the above two requirements.In meeting the requirements of standard printing, we found that the minimum order in this example is 44, which happens to be the number of layers of the printed object 43 plus 1.Under this order, each layer is a period. At the minimum order, you can see that the trajectory of the external axis is an almost perfect ellipse(You can see that at the beginning of the print, it's not perfect because of the Gibbs phenomenon).The comparison of different orders:

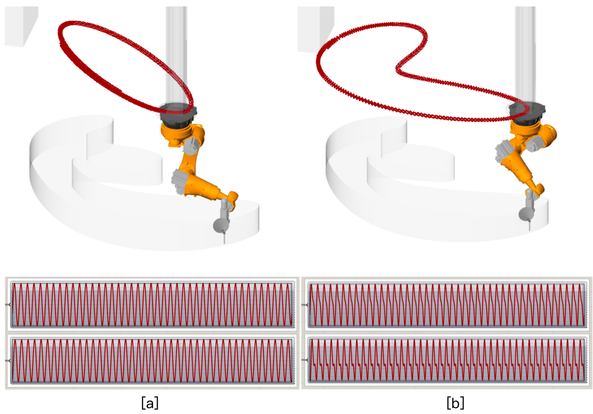

Figure 7. [a]:order=44, [b]:order=100.

When the order is 44, the external axis travels the least distance.In order to complete the same action, the robot needs to do more actions to compensate for the "lazy" external axis.At this time we will find that the A1 axis of the robot will rotate more than 180 degrees(Figure 8).For most robot, this is not a problem, because the limit of robot joint range can be lifted by setting.But if you encounter the problem of wire winding, you need to increase the order to avoid this problem.As long as the order is increased to a fixed value, the A1 axis of robot can keep stable in the whole process.

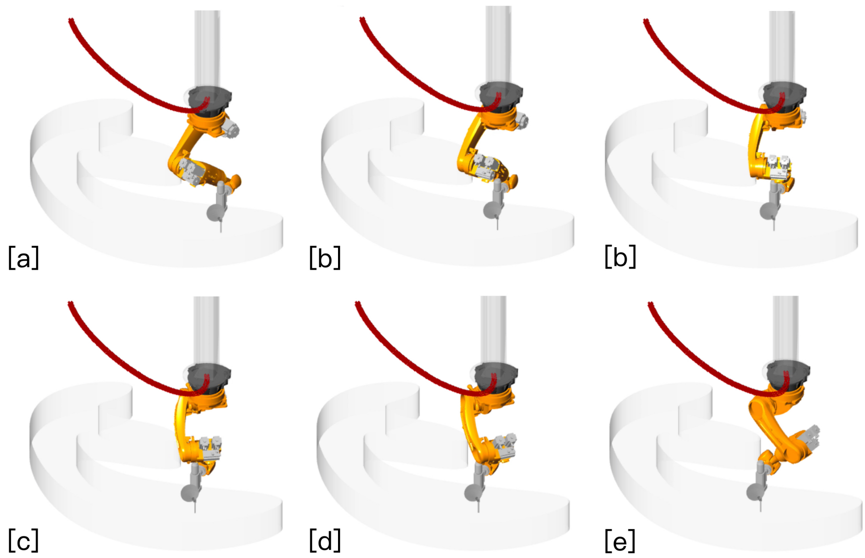

Figure 8. During the process from [a] to [e], the A1 axis rotated almost 180 degrees.

4. Conclusions

By introducing the Fourier series, this research contributes to the program of external axis movement. Printing stability and quality are improved with the proposed method. Furthermore, the proposed method is versatile and can be applied to various types of linear external axis programming.

- Add the acceleration control of the external axis to make the acceleration curve continuous,and control the acceleration value within a preset value(Spong,et al.2005)
- After obtaining the Fourier series, generally you can manually add an offset to each component so that the robot can print normally. This process can be improved by finding the position offset between the center of the target point cloud and the center of the external axis motion trajectory point cloud, and use this offset to replace the previous manual setting.

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